Form A

Math 2214 Common Part of Final Exam May 4, 1998

Instructions: Please enter your NAME, ID NUMBER, FORM designation, and INDEX NUMBER on your op-scan sheet. The index number should be written in the upper right-hand box labeled "Course." Do not include the course number. In the box labeled "Form," write the appropriate test form letter **A**. Darken the appropriate circles below your ID number and Form designation. **Use a #2 pencil; machine grading may ignore faintly marked circles.**

Mark your answers to the test questions in rows 1 - 14 of the op-scan sheet. You have 1 hour to complete this part of the final exam. Your score on this part of the final exam will be the number of correct answers. Please turn in your op-scan sheet and the question sheet at the end of this part of the final exam.

1. Consider the differential equation $\frac{dy}{dx} = x\,y^2$. If $y = 1$ when $x = 1$, then the value of y when $x = 0$ is:

(1) $\frac{2}{3}$ (2) $-\frac{1}{2}$ (3) $\frac{2}{5}$ (4) $\frac{5}{2}$

2. Consider the first order differential equation $y' = x + y + 1$ with initial condition $y(0) = 0$. If we use the Euler method with step size $h = 0.1$ to estimate $y(0.3)$, rounding off all calculations to two decimal places, the result will be:

(1) $y(0.3) = .22$ (2) $y(0.3) = .36$ (3) $y(0.3) = 0$ (4) $y(0.3) = .53$

3. The general solution of $y'' - y = e^t$ is:

(1) $y = C_1 \cos t + C_2 \sin t + 2e^t$ (2) $y = C_1 e^t + C_2 e^{-t} + \frac{1}{2} e^t$

(3) $y = C_1 e^t + C_2 e^{-t} + \frac{1}{2} t e^t$ (4) $y = C_1 \cos t + C_2 \sin t - t e^t$

4. The general solution of $y'' + y' + y = 0$ is:

(1) $y = C_1 e^{-\frac{1}{2}t} \cos \frac{\sqrt{3}}{2} t + C_2 e^{-\frac{1}{2}t} \sin \frac{\sqrt{3}}{2} t$ (2) $y = C_1 e^{-\frac{\sqrt{3}}{2}t} + C_2 e^{\frac{\sqrt{3}}{2}t}$

(3) $y = C_1 \cos \frac{\sqrt{3}}{2} t + C_2 \sin \frac{\sqrt{3}}{2} t$ (4) $y = C_1 \cos \frac{1}{2} t + C_2 \sin \frac{1}{2} t$

5. The eigenvalues of $\mathbf{A} = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ are $\lambda_1 = 5$ and $\lambda_2 = -1$ with corresponding eigenvectors $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ for $\lambda_1 = 5$ and $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ for $\lambda_2 = -1$. The solution of $\mathbf{x}' = \mathbf{A}\mathbf{x}$ which satisfies $\mathbf{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is:

(1) $\mathbf{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{5t}$

(2) $\mathbf{x} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} e^{5t} + \begin{pmatrix} -1 \\ -2 \end{pmatrix} e^{-t}$

(3) $\mathbf{x} = \begin{pmatrix} 1/3 \\ 1/3 \end{pmatrix} e^{5t} + \begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix} e^{-t}$

(4) $\mathbf{x} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{-t} + \begin{pmatrix} 3 \\ -1 \end{pmatrix} e^{5t}$

6. If the method of undetermined coefficients is used to find a particular solution y_p to the differential equation $y'' + 4y = 3t\sin 2t + 4e^{3t}$, the assumed form of y_p should be:

(1) $y_p = At\sin 2t + Bt\cos 2t + C\sin 2t + D\cos 2t + Ee^{3t}$

(2) $y_p = At\sin 2t + B\sin 2t + Ce^{3t}$

(3) $y_p = At^2\sin 2t + Bt^2\cos 2t + Ct\sin 2t + Dt\cos 2t + Ete^{3t}$

(4) $y_p = At^2\sin 2t + Bt^2\cos 2t + Ct\sin 2t + Dt\cos 2t + Ee^{3t}$

7. Consider the differential equation $y'' + 2y' - 3y = \sin t + \sec t$. If the method of variation of parameters is used to find a particular solution, the assumed form for the particular solution should be:

(1) $u_1(t)e^{-3t} + u_2(t)te^{t}$

(2) $u_1(t)e^{-3t} + u_2(t)(\sin t + \sec t)e^{t}$

(3) $u_1(t)\sin t + u_2(t)\sec t$

(4) $u_1(t)e^{-3t} + u_2(t)e^{t}$

8. Which of the following differential equations is linear?

(1) $y'' + 3yy' + 4y = 0$

(2) $y'' + ty^2 = t$

(3) $y'' + (t^2 + 1)y' + e^t y = \sin t$

(4) $y'' + 2y' + y = \cos y$

9. At initial time t = 0, a tank contains 100 gallons of pure water. Then water containing 1 lb. of salt per gallon flows into the tank at the rate of 5 gallons per minute, and the well-stirred mixture leaves the tank at the same rate. Let $Q(t)$ denote the number of pounds of salt in the tank after t minutes.Then when $t \geq 0$, Q satisfies the differential equation and initial condition:

(1) $Q' = 5 - \frac{Q}{20}; \quad Q(0) = 0$

(2) $Q' = 5 - \frac{Q}{100}; \quad Q(0) = 0$

(3) $Q' = 5Q - 5; \quad Q(0) = 100$

(4) $Q' = 1 - \frac{Q}{100}; \quad Q(0) = 100$

10. The solution of $y'' - 4y' + 4y = 0$ satisfying $y(0) = 0$, $y'(0) = 2$, is:

(1) $y = e^{2t} - 1$

(2) $y = 2te^{2t}$

(3) $y = \frac{1}{2}e^{2t} - \frac{1}{2}e^{-2t}$

(4) $y = e^{2t} - te^{2t}$

11. Consider the first order linear differential equation $t y' + y = \tan t$ with initial condition $y(1) = 1$. The largest interval on which a unique solution is guaranteed to exist is:

(1) $(-\infty, \frac{\pi}{2})$

(2) $(1, \frac{\pi}{2})$

(3) $(0,1)$

(4) $(0, \frac{\pi}{2})$

12. The general solution of $t^2 y' + 2ty = e^t$ is:

(1) $y = C_1 e^t + \frac{1}{t^2}$

(2) $y = \frac{1}{t^2}(e^t + C_1)$

(3) $y = t^2 e^t + C_1 \frac{1}{t^2}$

(4) $y = C_1 e^t + C_2 \frac{1}{t^2}$

13. If $y_1(t) = t^2$ is one solution to the differential equation $t^2y'' - 4t\,y' + 6y = 0$, then the solution y(t) for which $y(1) = 0$ and $y'(1) = 2$ satisfies:

(1) $y(0) = 0$ (2) $y(0) = 1$ (3) $y(0) = 8$ (4) $y(0) = -\frac{1}{2}$

14. The matrix $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 4 & 6 \end{pmatrix}$ has one eigenvector $\mathbf{v} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ for the repeated eigenvalue $\lambda = 4$, and it has a generalized eigenvector $\mathbf{w} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ satisfying $(\mathbf{A} - 4\mathbf{I})\mathbf{w} = \mathbf{v}$. The general solution of $\mathbf{x}' = \mathbf{A}\mathbf{x}$ is:

(1) $\mathbf{x} = C_1 e^{4t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{4t}\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

(2) $\mathbf{x} = C_1 e^{4t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{4t}\left[t\begin{pmatrix} 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 1 \\ -3 \end{pmatrix} \right]$

(3) $\mathbf{x} = C_1 e^{4t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 t e^{4t}\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

(4) $\mathbf{x} = C_1 e^{4t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{4t}\left[t^2\begin{pmatrix} 1 \\ -3 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \end{pmatrix} \right]$