Form A

Math 2214 Common Part of Final Exam May 10, 1999

INSTRUCTIONS:

Please enter your NAME, ID NUMBER, FORM designation, and INDEX NUMBER on your op-scan sheet. The index number should be written in the upper right-hand box labeled "Course". In the box labeled "Form", write the appropriate test form letter (A, B, or C). Darken the appropriate circles below your ID number and Form designation. **Use a #2 pencil**.

Mark your answers to the test questions in rows 1-12 of the op-scan sheet. You have 1 hour to complete this part of the final exam. Your score on this part of the final exam will be the number of correct answers. Please turn in the op-scan sheet with your answers and this question sheet at the end of this part of the final exam.

NOTE WELL: Throughout the exam arbitrary constants are denoted by c, c_1 or c_2 .

Your signature is required on this form:

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Signature

1. If $y' + y + 1 = 0$ and $y(0) = 0$, then $y(1) =$

(1) $-1 + (1/e)$
(2) $-1 - (1/e)$
(3) $(1/e)$
(4) e

2. Given two solutions $y_1(t)$ and $y_2(t)$ of a homogeneous linear differential equation, consider the following statements:

(a.) $y_1(t) + y_2(t)$ *must* also be a solution to the differential equation.
(b.) $2y_1(t)$ *must* also be a solution to the differential equation.
(c.) $y_1(t) \cdot y_2(t)$ *must* also be a solution to the differential equation.
(d.) $6y_1(t) - y_2(t)$ *must* also be a solution to the differential equation.

Which of the statements are always true?
(1) (a.) only
(2) (a.) and (b.) only
(3) (a.), (b.) and (c.) only
(4) (a.), (b.) and (d.) only
(5) all are true

3. If $y' = y^2 \sin(t)$, and $y(0) = -1$, then $y\left(\frac{p}{2}\right) =$

(1) -2
(2) y is undefined at $\left(\frac{p}{2}\right)$.
(3) 2
(4) $-1/2$

4. At time $t = 0$, a tank contains 500 gallons of well-mixed salt water containing 100 lbs of salt. Pure water enters the tank at 10 gallons per minute and the well-stirred mixture leaves at the same rate. Let $Q(t)$ be the number of pounds of salt in the tank after t minutes. Then

(1) $Q(t) = e^{-t/50}$
(2) $Q(t) = 2 + 98e^{-t/50}$
(3) $Q(t) = 100e^{-t/50}$
(4) $Q(t) = 500 - 400e^{-t/50}$

5. A third order linear homogeneous constant coefficient differential equation has characteristic equation $(r-1)^2(r-3)=0$. The general solution of the differential equation has the form

(1) $y=c_1e^t+c_2e^t+c_3e^{3t}$
(2) $y=c_1(1+t)e^t+c_2e^{3t}$
(3) $y=c_1e^t+c_2te^t+c_3e^{3t}$
(4) $y=c_1t+c_2t^2+c_3e^{3t}$

6. The general solution of $y''-2y'+17y=0$ has the form

(1) $y=c_1\cos(4t)+c_2\sin(4t)$
(2) $y=c_1e^t\cos(4t)+c_2e^t\sin(4t)$
(3) $y=c_1t\cos(4t)+c_2t\sin(4t)$
(4) $y=c_1e^{4t}\cos(t)+c_2e^{4t}\sin(t)$

7. Given that the homogeneous differntial equation $y''-9y'+20y=0$ has general solution $c_1e^{5t}+c_2e^{4t}$, find the solution y to the initial value problem $y''-9y'+20y=24e^t$, $y(0)=3$, $y'(0)=6$ is

(1) $y=e^{4t}+2e^t$
(2) $y=9e^{4t}-6e^{5t}+2e^t$
(3) $y=9e^{4t}-6e^{5t}$
(4) $y=3e^{2t}$

8. The second order linear homogeneous constant coefficient differential equation $y''+y'-6y=0$ has general solution $y=c_1e^{2t}+c_2e^{-3t}$. Which of the following statements describes the behavior of the solution of the initial value problem $y''+y'-6y=0$, $y(0)=a$, $y'(0)=3$?

(1) For every value of α, the solution does not approach 0 as $t\to\infty$.
(2) For every value of α, the solution approaches 0 as $t\to\infty$.
(3) The solution approaches 0 as $t\to\infty$ if and only if $a=\frac{3}{2}$.
(4) The solution approaches 0 as $t\to\infty$ if and only if $a=-1$.

9. An equation of the form $u''' + au'' + bu' + cu = 0$ is transformed to a system of first order equations of the form $\mathbf{x}' = A\mathbf{x}$ where the matrix A is given by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & 6 & 5 \end{bmatrix}$$

Then

(1) $a = -5; b = -6; c = 4$
(2) $a = 5; b = 6; c = -4$
(3) $a = -4; b = 6; c = 5$
(4) $a = 4; b = -6; c = -5$

10. Consider an initial value problem of the form $\mathbf{x}' = A\mathbf{x}$, $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. For which matrix A does $\lim_{t\to\infty} \mathbf{x}(t) = \mathbf{0}$?

(1) $A = \begin{bmatrix} -1 & 3 \\ 0 & -3 \end{bmatrix}$

(2) $A = \begin{bmatrix} 1 & 3 \\ 0 & 3 \end{bmatrix}$

(3) $A = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

(4) $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

11. A 2×2 matrix A has eigenvalues $l_1 = 3$ and $l_2 = 7$. An eigenvector corresponding to $l_1 = 3$ is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$. An eigenvector corresponding to $l_2 = 7$ is $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$. The general solution of $\mathbf{x}' = A\mathbf{x}$ is given by

(1) $c\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{3t} + \begin{bmatrix} 2 \\ 5 \end{bmatrix} e^{7t}\right)$

(2) $c_1 e^{3t} + c_2 e^{7t}$

(3) $c_1 \begin{bmatrix} 2 \\ 4 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} 4 \\ 10 \end{bmatrix} e^{7t}$

(4) $c_1 \begin{bmatrix} 3 \\ 7 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} -1 \\ 3 \end{bmatrix} e^{7t}$

12. Let $A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$. The vector $\mathbf{w} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ is an eigenvector for *A*. A corresponding eigenvalue is

(1) $l = -1$
(2) $l = -2$
(3) $l = 2$
(4) $l = 4$