Form A

Math 2214 Common Part of Final Exam Dec 13, 1999

INSTRUCTIONS: Please enter your NAME, ID NUMBER, FORM designation, and INDEX NUMBER on your op-scan sheet. The index number should be written in the upper right-hand box labeled "Course". In the box labeled "Form", write the appropriate test form letter **A**. Darken the appropriate circles below you ID number and Form designation. **Use a #2 pencil.**

Mark your answers to the test questions in rows 1-14 of the op-scan sheet. You have 1 hour to complete this part of the final exam. Your score on this part of the final exam will be the number of correct answers. Please turn in the op-scan with your answers and this question sheet at the end of this part of the final exam.

1. Four solutions of $y'''' - y = 0$ are?
   (a) $e^t, te^t, t^2e^t, t^3e^t$
   (b) $e^t\cos t, e^t\sin t, te^t\cos t, te^t\sin t$
   (c) $e^t, te^t, e^{-t}, te^{-t}$
   (d) $e^t, e^{-t}, \sin t, \cos t$

2. The general solution of $y'' + 5y' + 4y = 0$ is?
   (a) $y = c_1e^{4t} + c_2e^t$
   (b) $y = c_1e^{5t} + c_2e^{4t}$
   (c) $y = c_1e^{-4t} + c_2e^{-t}$
   (d) $y = c_1e^{-5t} + c_2e^{-4t}$

3. A particular solution for the differential equation $y'' + 25y = 6\sin t$ would be
   (a) $y_p = c_1\cos 5t + c_2\sin 5t$
   (b) $y_p = \frac{1}{4}\sin t$
   (c) $y_p = \frac{2}{5}\cos t$
   (d) $y_p = \frac{2}{5}\cos t + \frac{1}{4}\sin t$
   (e) $y_p = \frac{1}{4}\cos 5t + \frac{2}{5}\sin 5t$

4. The third order differential equation $y'''+10y''-9y'+3y=\cos t$ can be rewritten as the first order system ______.

(a) $\begin{cases} x_1' = 3x_2 \\ x_2' = \dfrac{10}{9}x_3 \\ x_3' = -10x_3 + 9x_2 - 3x_1 + \cos t \end{cases}$

(b) $\begin{cases} x_1' = x_2 \\ x_2' = x_3 \\ x_3' = -10x_3 + 9x_2 - 3x_1 + \cos t \end{cases}$

(c) $\begin{cases} x_1' = x_2 \\ x_2' = x_3 \\ x_3' = -10x_1 + 9x_2 - 3x_3 + \cos t \end{cases}$

(d) $\begin{cases} x_1' = x_2 \\ x_2' = x_3 \\ x_3' = 3x_1 - 9x_2 + 10x_3 + \cos t \end{cases}$

5. A linear system of two first order differential equations with real coefficients has the complex solution $X(t) = e^{(2+3i)t}\begin{bmatrix} i \\ 1 \end{bmatrix}$. Then two real-valued solutions are given by

(a) $e^{2t}\begin{bmatrix} -\sin 3t \\ \cos 3t \end{bmatrix}, e^{2t}\begin{bmatrix} \cos 3t \\ \sin 3t \end{bmatrix}$

(b) $e^{3t}\begin{bmatrix} -\sin 2t \\ \cos 2t \end{bmatrix}, e^{3t}\begin{bmatrix} \cos 2t \\ \sin 2t \end{bmatrix}$

(c) $\begin{bmatrix} -\sin 3t \\ \cos 3t \end{bmatrix}, \begin{bmatrix} \cos 3t \\ \sin 3t \end{bmatrix}$

(d) $\begin{bmatrix} \cos 3t - \sin 3t \\ \cos 3t + \sin 3t \end{bmatrix}, \begin{bmatrix} \sin 3t - \cos 3t \\ -\sin 3t - \cos 3t \end{bmatrix}$

6. Use Euler's Method with a step size of $h = 0.5$ to approximate the value of $y(1)$ given the initial value problem $y' = 3t^2 - 2y, y(0) = 0$.

(a) $y(1) \approx 0$
(b) $y(1) \approx 0.375$
(c) $y(1) \approx 0.5$
(d) $y(1) \approx 0.75$

7. Given that $y_1 = t$ and $y_2 = \frac{1}{t}$ are solutions of the homogeneous differential equation $t^2 y'' + ty' - y = 0$ and the function $y_p = t^2$ is a particular solution of the nonhomogeneous differential equation $t^2 y'' + ty' - y = 3t^2$. The general solution of the nonhomogeneous differential equation is

(a) $y = t + \frac{1}{t} + t^2$

(b) $y = c_1 t + c_2 \frac{1}{t} + c_3 t^2$

(c) $y = c_1 t + c_2 \frac{1}{t} + 3t^2$

(d) $y = c_1 t + c_2 \frac{1}{t} + t^2$

8. The general solution of the system $X'(t) = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} X$ is

(a) $c_1 e^{3t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 e^{-t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(b) $c_1 e^{3t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 e^{-t} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

(c) $c_1 e^{3t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 e^{-t} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(d) $c_1 e^{3t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 e^{-t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

9. The solution of the initial value problem $y'' - y = e^t$ with $y(0) = 2, y'(0) = 1$ is

(a) $\frac{5}{4} e^t + \frac{3}{4} e^{-t} + \frac{1}{2} te^t$

(b) $\frac{3}{2} e^t + \frac{1}{2} e^{-t} + \frac{1}{2} te^t$

(c) $2\cos t + \sin t + \frac{1}{2} te^t$

(d) $2e^t - \frac{3}{2} e^{-t} + \frac{1}{2} te^t$

10. Which of the following differential equations is <u>not</u> linear?

(a) $y''' + ty' + (\cos^2 t) y = t^3$

(b) $y''' + 3y'' + 4y' - 8y^2 = 0$

(c) $(1 + x^2) y'' + e^{-x} y' + y = e^x$

(d) $u''(t) + 2u'(t) + 5u(t) = 3\cos 2t$

11. Find an implicit solution of the differential equation $y' = \frac{\cos t}{5y^4}$.

(a) $y^5 = \sin t + C$
(b) $y^5 + \sin t = C$
(c) $y^5 = C \sin t$
(d) $y = \frac{\sin t}{y^5} + C$

12. The solution of the initial value problem $ty' - y = \frac{t^3}{t^2+1}, y(1) = 0$ is

(a) $\frac{1}{2} t \ln(t^2+1) - (\frac{1}{2}\ln 2)t$
(b) $t^2 - t\tan^{-1} t + (\frac{\pi}{4} - 1)t$
(c) $-\frac{1}{2} t \ln(t^2+1) - t\ln t + (\frac{1}{2}\ln 2)t$
(d) $t\tan^{-1} t - \frac{\pi}{4} t$

13. The decay constant for thorium-234 is 0.02828 days^{-1}. The differential equation which governs the decay of thorium-234 when the isotope is being replenished at a rate of 2 mg/day is

(a) $\frac{dQ}{dt} = 2 - 0.02828 Q(t)$
(b) $\frac{dQ}{dt} = -0.02828 + 2Q(t)$
(c) $\frac{dQ}{dt} = 0.02828 Q(t) - 2$
(d) $\frac{dQ}{dt} = 0.02828 - 2Q(t)$

14. Matrix $A = \begin{bmatrix} 2 & -1 \\ 4 & 6 \end{bmatrix}$ has only one eigenvector $v = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ for the repeated eigenvalue $\lambda = 4$. Also, a solution of $[A - \lambda I]\omega = v$ is $\omega = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$. Then general solution of $x' = Ax$ is

(a) $x = c_1 e^{4t}\begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 e^{4t}\begin{bmatrix} 1 \\ -3 \end{bmatrix}$
(b) $x = c_1 e^{4t}\begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 t e^{4t}\begin{bmatrix} 1 \\ -3 \end{bmatrix}$
(c) $x = c_1 e^{4t}\begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 e^{4t}\left\{ t\begin{bmatrix} 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix} \right\}$

(d) $x = c_1 e^{4t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 e^{4t} \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix} + t \begin{bmatrix} 1 \\ -3 \end{bmatrix} \right\}$