Form A

Math 2214 Common Part of Final Exam May 8, 2000

Instructions: Please enter your NAME, ID NUMBER, FORM designation, and INDEX NUMBER on your op-scan sheet. The index number should be written in the upper right-hand box labeled "Course." Do not include the course number. In the box labeled "Form," write the appropriate test form letter **C.** Darken the appropriate circles below your ID number and Form designation. **Use a #2 pencil; machine grading may ignore faintly marked circles.**

Mark your answers to the test questions in row 1-15 of the op-scan sheet. You have 1 hour to complete this part of the final exam. Your score on this part of the final exam will be the number of correct answers. Please turn in your op-scan sheet and the question sheet at the end of this part of the final exam.

1. The differential equation

$$e^t\, y'' + (\sin\, t)\, y = 1 \qquad \text{is}$$

(a) nonlinear (b) homogeneous (c) linear (d) separable

2. A tank initially contains 200 liters of pure water. A mixture containing a concentration of 2 g/liter of salt enters the tank at a rate of 2 liters/min, and the well-stirred mixture leaves the tank at the same rate. Let $Q(t)$ denote the amount of salt in the tank at time t. Then $Q(t)$ is a solution of the initial value problem

(a) $Q'(t) = 2 - 2\,Q(t), \qquad Q(0) = 200$

(b) $Q'(t) = 2 - \dfrac{Q(t)}{200}, \qquad Q(0) = 0$

(c) $Q'(t) = 4 - \dfrac{Q(t)}{100}, \qquad Q(0) = 0$

(d) $Q'(t) = 2 - \dfrac{Q(t)}{100}, \qquad Q(0) = 0$

3. Suppose that the functions e^{2t} and e^{-3t} form a fundamental set of solutions for the differential equation

$$y'' + a_1\,y' + a_0\,y = 0 \qquad (a_0, a_1 \quad \text{constants}).$$

Then the coefficient a_0 is

(a) 1 (b) 3 (c) -2 (d) -6

4. Let $Q(t)$ denote the amount of a certain radioactive material (measured in mg) at time t measured in years. If $Q(10) = 18$ mg and $Q(20) = 8$ mg, find $Q(5)$. Assume that decay occurs naturally, and no new material is added.

(a) 20 mg (b) 27 mg (c) 24 mg (d) 36 mg

5. The general solution of the differential equation

$$t\,y' + y = t^2 \quad (t > 0) \qquad \text{is}$$

(a) $\dfrac{t^4}{4} + \dfrac{C}{t}$ (b) $\dfrac{t^2}{3} + C$ (c) $\int_0^t e^{\ln s}\,ds + C$ (d) $\dfrac{t^2}{3} + \dfrac{C}{t}$

6. The solution of the initial value problem

$$y' = \frac{-e^{-t}}{y}, \qquad y(0) = 1, \qquad \text{is}$$

(a) $y = e^{-t}\ln y + 1$ (implicit form)

(b) $\sqrt{2e^{-t} - 1}$

(c) $\sqrt{-2e^{-t} + 3}$

(d) $\sqrt{2}\, e^{-t/2} + 1 - \sqrt{2}$

7. The differential equation $2\,y'' - 3\,y' + 4\,y = e^{5t}$ can be converted to which first order system:

(a) $$\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & \frac{3}{2} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 \\ e^{5t} \end{bmatrix}$$

(b) $$\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{3}{2} & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \frac{1}{2} \begin{bmatrix} e^{5t} \\ 0 \end{bmatrix}$$

(c) $$\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \frac{1}{2} \begin{bmatrix} e^{5t} \\ 0 \end{bmatrix}$$

(d) $$\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 \\ e^{5t} \end{bmatrix}$$

8. Suppose that a differential equation of the form

$$a\,y'' + b\,y' + c\,y = 0 \qquad (a, b, c \quad \text{real constants})$$

has the solution $y(t) = e^t \cos 2t$. Then a second, linearly independent solution is given by

(a) $e^{-t} \cos 2t$

(b) $-e^t \cos 2t$

(c) $3\,e^t \sin 2t + 2\,e^t \cos 2t$

(d) $e^{-2t} \sin t$

9. Consider the initial value problem

$$y' = t^2\,y^3, \qquad y(2) = -1.$$

Use the Euler method taking one step to calculate an approximation of $y(2.3)$. The approximate value of $y(2.3)$ is

(a) 2.8 (b) -1.2 (c) 1.4 (d) -2.2

10. The Wronskian of the two solutions $y_1(t) = e^{2t}$ and $y_2(t) = t\,e^{2t}$ of the differential equation $y'' - 4\,y' + 4\,y = 0$ is

(a) e^{4t}

(b) $4\,t\,e^{4t}$

(c) $e^{4t} + 4\,t\,e^{4t}$

(d) $t - \frac{1}{2}\,e^{4t} - \frac{t}{2}\,e^{4t}$

11. Determine (without solving the differential equation) an interval on which the solution to the initial value problem

$$y' + \frac{t}{t-1}\, y = t^2, \qquad y\left(\frac{1}{2}\right) = 2,$$

is certain to exist:

(a) $(-2, 2)$ (b) $(-1, \infty)$ (c) $(-\infty, 1)$ (d) $(-\infty, \infty)$

12. The general real valued solution to

$$y''' - 2\, y'' + y' = 0 \qquad \text{is}$$

(a) $c_1 + c_2\, e^{-t} + c_3\, t\, e^{-t}$

(b) $c_1 + c_2\, e^{t} + c_3\, t\, e^{t}$

(c) $c_1\, t + c_2\, e^{t} + c_3\, t\, e^{t}$

(d) $c_1\, t + c_2\, e^{-t} + c_3\, t\, e^{-t}$

13. The solution to the initial value problem

$$x' = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} x, \qquad x(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \qquad \text{is}$$

(a) $e^{-t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(b) $e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^{3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(c) $e^{t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + e^{-3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(d) $e^{t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^{3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

14. If

$$x(t) = c_1 e^t \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^t \left\{ t \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/2 \end{bmatrix} \right\}$$

is the general solution of the system $x' = A\,x$, where A is a 2×2 matrix with real constant coefficients, then

(a) A has two distinct real eigenvalues

(b) A has a repeated real eigenvalue

(c) A has a pair of distinct complex conjugate eigenvalues

(d) A has a repeated nonreal (complex) eigenvalue

15. A suitable form for a particular solution of the differential equation

$$y'' + 3\,y' + 2\,y = e^{-t} + e^{-2t}$$

is

(a) $A\,e^{-t} + B\,t\,e^{-2t}$

(b) $A\,t\,e^{-t} + B\,e^{-2t}$

(c) $A\,e^{-t} + B\,e^{-2t}$

(d) $A\,t\,e^{-t} + B\,t\,e^{-2t}$